Stats 579 – Intermediate Bayesian Modeling

Assignment # 2

1. Let $w \equiv G(y)$ with y a vector having density $f(y \mid \theta)$ and G having a differentiable inverse function. Find the density of w in general and show that the likelihoods satisfy $L(\theta \mid y) \propto L(\theta \mid w)$. (HINT: Proposition B.4 from the textbook may be useful.)

2. Let $y_i \sim f(y_i \mid \theta)$ where $i \in \{1, ..., n\}$, and let $\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$. Prove the following statement:

$$\sum_{i=1}^{n}(y_i - \theta)^2 = \sum_{i=1}^{n}(y_i - \overline{y})^2 + n(\overline{y} - \theta)^2$$

3. Let $y|\theta \overset{\text{iid}}{\sim} \text{Exp}\,(\theta)$ where $i \in \{1, ..., n+1\}$, and let $p(\theta) = e^{-\theta}$.

   (a) Given $y = \{y_1, ..., y_n\}$, obtain the predictive probability that $y_{n+1} > t_0$ using calculus. Argue that this probability is the posterior mean of a particular function of θ.

   (b) How would you interpret the difference between the meaning of these two quantities (the predictive probability and the posterior mean of a function of θ), despite the fact that the values are identical?

4. Let $y|\theta \overset{\text{iid}}{\sim} \text{Pois}\,(\theta)$ where $i \in \{1, ..., n+1\}$, and let $p(\theta) = e^{-\theta}$. Given $y = \{y_1, ..., y_n\}$, obtain the predictive probability that $y_{n+1} = 0$ using calculus. Argue that this probability is the posterior mean of a particular function of θ.