1. Consider the bivariate normal population with \( \mu_1 = 0, \mu_2 = 2, \sigma_{11} = 2, \sigma_{22} = 1, \) and \( \rho_{12} = .5. \)

(a) Write out the bivariate normal density.
(b) Write out the squared statistical distance expression \((x - \mu)'\Sigma^{-1}(x - \mu)\) as a function of \(x_1\) and \(x_2\).
(c) Determine and sketch the constant contour that contains 90% of the probability.

2. Let \( X \sim \mathcal{N}(\mu, \Sigma) \) with \( \mu' = (2, -3, 1, 4) \) and

\[
\Sigma = \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 4 & 3 & 2 \\
1 & 3 & 3 & 2 \\
1 & 2 & 2 & 2 \\
\end{pmatrix}
\]

(a) Find the distribution of \(3X_1 - 2X_2 + X_3\).
(b) Are \(X_1\) and \(X_3 - X_4\) independent? Is \((X_1, X_2)\) independent of \(X_3 - X_4\)? Why or why not?
(c) Let \( A \) be a matrix whose rows are the eigenvectors of \( \Sigma \). What is the distribution of \( Y = AX? \) What do you notice? Why is this the case? (Hint: think of the spectral representation of \( \Sigma \).)
(d) Find the conditional distribution of \(X_4\) given \((X_1 = 1.5, X_2 = 0, X_3 = 2)\).
(e) Find the conditional distribution of \((X_2, X_3)\) given \((X_1 = 1.5, X_4 = 2)\).
(f) Generate a random sample of size \(n = 100\) from this distribution. Calculate \(\bar{x}\) and \(S\).

3. From book problem 4.39. The data at “http://www.stat.unm.edu/~storlie/st576/hw2_data.txt” consists of 130 observations generated by scores on a psychological test administered to Peruvian teenagers (ages 15 - 17). For each of these teenagers, the gender (male=1, female=2) and socioeconomic status (low=1, medium=2) were also recorded. The scores were accumulated into five subscale scores labeled independence (Indep), support (Supp), benevolence (Benev), Conformity (Conform), and leadership (Leader).

Recall you can use
\[
> \text{mydata} <- \text{read.table(file="http://www.stat.unm.edu/~storlie/st576/hw2_data.txt", header=TRUE)}
\]
to read the data into an R data frame.

(a) Examine each of the five variables for marginal normality. Comment.
(b) Examine all pairwise scatterplots for bivariate normality. Comment.
(c) Calculate the statistical distance from each observation to the mean. Comment on any outliers.
(d) Are transformations necessary to any of the variables to help satisfy the normality assumption? If so, apply appropriate transformations and verify that the normality assumption is feasible for the transformed data.