Examples of Complex Analyses Involving a Separation of Aleatory and Epistemic Uncertainty

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Antecedents to the Quantification of Margins and Uncertainty: QMU Is Not New

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Quantification of Margins and Uncertainties is a scientific methodology that identifies relevant nuclear-warhead parameters and quantifies, using available experimental and computational tools, the margin of that parameter relative to its failure point and the uncertainties associated with the parameter and the failure point. An assessment of the relationship between the margin and uncertainties facilitates stockpile management decisions, resource allocation prioritization, and informed judgments on the safety, reliability and performance of nuclear warheads.

Uncertainty is a best estimate of the range of a particular metric which may derive from one or two broad sources. Uncertainties that reflect a lack of knowledge about the appropriate value to use for a quantity that is assumed to have value in the context of a particular analysis are termed epistemic. Uncertainties that arise from an inherent randomness in the behavior of the system under study are termed aleatoric.
Two Distinct Applications of QMU

- Comparison of experimental results against a requirement without the use of a mathematical model to transform the experimental results

- Comparison of predictions from a mathematical model against a requirement

- This presentation treats the second application: Comparison of predictions from a mathematical model against a requirement
Two Views of QMU for a Systems-Level Analysis

• Informal view: Four questions
  – What can happen?
  – How likely is it to happen?
  – What are the consequences if it does happen?
  – How much confidence exists in the answers to the preceding questions?

• Formal view: Three basic components
  – Probability space characterizing aleatory uncertainty
  – Model that represents physical behavior of system
  – Probability space characterizing epistemic uncertainty
Antecedents to QMU: Three Examples

- Reactor accident safety goals
- Regulatory requirements for Waste Isolation Pilot Plant (WIPP)
- Regulatory requirements for Yucca Mountain (YM) waste repository
Ex. 1: Reactor Accident Safety Goals Proposed by NRC

- **Individual early fatality risk**: the expected value for average individual early fatality risk in the region between the plant site boundary and 1 mi beyond this boundary will be less than $5 \times 10^{-7}$ yr$^{-1}$. (SG1)

- **Individual latent cancer fatality risk**: the expected value for average individual latent cancer fatality risk in the region between the plant site boundary and 10 mi beyond this boundary will be less than $2 \times 10^{-6}$ yr$^{-1}$. (SG2)

- **Severe accident frequency**: the expected value for the frequency of a severe accident will be less than $1 \times 10^{-4}$ yr$^{-1}$. (QRG1)

- **Conditional probability of containment failure**: the expected value for the probability of containment failure given the occurrence of a severe accident will be less than 0.1. (QRG2)

- **Large release frequency**: the expected value for the frequency of a large release will be less than $1 \times 10^{-6}$ yr$^{-1}$. (QRG3)
Ex. 1: NUREG-1150 PRA for Surry Nuclear Power Station

- **Probability space** \((\mathcal{A}, \mathbb{A}, p_A)\) for aleatory uncertainty:
  - \(\mathcal{A} = \{a: a = [IE, AS, PDS, APB, STG, WT]\}\)
    where
    - \(IE\) = designator for initiating event \((nIE = 11)\),
    - \(AS\) = designator for accident sequence \((nAS = 28)\),
    - \(PDS\) = designator for plant damage state \((nPDS = 25 \Rightarrow nPDS = 7)\),
    - \(APB\) = designator for accident progression bin \((54 \leq nAPB \leq 157)\),
    - \(STG\) = designator for source term group \((nSTG = 54)\),
    - \(WT\) = designator for weather type \((nWT = 2560)\).
  - \((\mathcal{A}, \mathbb{A}, p_A)\) developed with extensive use of fault and event trees

- **Probability space** \((\mathcal{E}, \mathbb{E}, p_E)\) for epistemic uncertainty
  - \(\mathcal{E} = \{e: e = [e_1, e_2, ..., e_{130}]\}\)
  - \((\mathcal{E}, \mathbb{E}, p_E)\) developed through extensive expert review process
  - Implemented with LHS of size 200

- Model \(y = f(a|e)\) representing physical behavior of system: Combination of models for physical processes (e.g., STCP, CONTAIN, MELCOR, MACCS) and interpolation/extrapolation procedures
Ex. 1: Individual Early Fatality Risk for Surry

- **Individual early fatality risk**: the expected value for average individual early fatality risk in the region between the plant site boundary and 1 mi beyond this boundary will be less than $5 \times 10^{-7}$ yr$^{-1}$.

\[
\begin{align*}
\text{mSG1}_{0.5} &= 4.99 \times 10^{-7} \text{ yr}^{-1} \\
\text{mSG1}_{mn} &= 4.84 \times 10^{-7} \text{ yr}^{-1} \\
\text{mSG1}_{0.05} &= 4.61 \times 10^{-7} \text{ yr}^{-1}
\end{align*}
\]

\[
\begin{align*}
mSG1_{mn}/(mSG1_{mn} - mSG1_{0.05}) &= 4.84 \times 10^{-7}/(4.84 \times 10^{-7} - 4.61 \times 10^{-7}) \\
&= 21.0
\end{align*}
\]

\[
\begin{align*}
mSG1_{0.5}/(mSG1_{0.5} - mSG1_{0.05}) &= 4.99 \times 10^{-7}/(4.99 \times 10^{-7} - 4.61 \times 10^{-7}) \\
&= 13.1
\end{align*}
\]
Ex. 1: Individual Latent Cancer Fatality Risk for Surry

- **Individual latent cancer fatality risk**: the expected value for average individual latent cancer fatality risk in the region between the plant site boundary and 10 mi beyond this boundary will be less than \(2 \times 10^{-6} \text{ yr}^{-1}\). (SG2)

\[
\begin{align*}
\text{mSG2}_{0.5} & = 1.9995 \times 10^{-6} \\
\text{mSG2}_{mn} & = 1.9983 \times 10^{-6} \\
\text{mSG2}_{0.05} & = 1.9921 \times 10^{-6}
\end{align*}
\]

\[
\begin{align*}
\text{mSG2}_{mn} / (\text{mSG2}_{mn} - \text{mSG2}_{0.05}) & = 1.9983 \times 10^{-6} / \left(1.9983 \times 10^{-6} - 1.9921 \times 10^{-6}\right) \\
& = 322.3 \\
\text{mSG2}_{0.5} / (\text{mSG2}_{0.5} - \text{mSG2}_{0.05}) & = 1.9995 \times 10^{-6} / \left(1.9995 \times 10^{-6} - 1.9921 \times 10^{-6}\right) \\
& = 270.2
\end{align*}
\]
Ex. 1: Severe Accident Frequency for Surry

- **Severe accident frequency**: the expected value for the frequency of a severe accident will be less than $1 \times 10^{-4}$ yr$^{-1}$.

\[
m_{QRG1_{0.5}}/(m_{QRG1_{0.5} - m_{QRG1_{0.05}}}) = 5.94 \times 10^{-5}/(5.94 \times 10^{-5} - 3.28 \times 10^{-7}) = 1.01
\]

\[
m_{QRG1_{0.05}}/(m_{QRG1_{0.05} - m_{QRG1_{0.05}}}) = 7.46 \times 10^{-5}/(7.46 \times 10^{-5} - 3.28 \times 10^{-7}) = 1.00
\]
Ex. 1: Conditional Probability of Containment Failure for Surry

- **Conditional probability of containment failure**: the expected value for the probability of containment failure given the occurrence of a severe accident will be less than 0.1. (QRG2)

\[
mQRG_{2_{mn}}/(mQRG_{2_{mn}} - mQRG_{2_{0.05}}) = 9.36 \times 10^{-2}/(9.36 \times 10^{-2} - 7.46 \times 10^{-2})
\]
\[
= 4.9
\]
\[
mQRG_{2_{0.5}}/(mQRG_{2_{0.5}} - mQRG_{2_{0.05}}) = 1.00 \times 10^{-2}/(1.00 \times 10^{-2} - 7.46 \times 10^{-2})
\]
\[
= 3.9
\]
Ex. 1: Large Release Frequency for Surry

- **Large release frequency**: the expected value for the frequency of a large release will be less than $1 \times 10^{-6}$ yr$^{-1}$.

\[
mQRG3 = 1 \times 10^{-6} \text{yr}^{-1}
\]

\[
mQRG3_{0.5}/(mQRG3_{mn} - mQRG3_{0.05}) = \begin{cases} 
2.4 & \text{for 1EF} \\
0.65 & \text{for 0.1EF} \\
0.17 & \text{for 0.01EF}
\end{cases}
\]

\[
mQRG3_{0.5}/(mQRG3_{0.5} - mQRG3_{0.05}) = \begin{cases} 
2.0 & \text{for 1EF} \\
0.73 & \text{for 0.1EF} \\
0.36 & \text{for 0.01EF}
\end{cases}
\]
Ex. 2: Regulatory Requirements for WIPP

- Paraphrase of EPA’s core requirement in 40 CFR 191, Subpart B, for WIPP:
  The probability of exceeding a normalized release of size $R = 1$ over $10^4$ years must be less than 0.1 (RR1)
  The probability of exceeding a normalized release of size $R = 10$ over $10^4$ years must be less than $10^{-3}$. (RR2)

- Additional EPA guidance in 40 CFR 194

  § 194.34 Results of performance assessments.
  (a) The results of performance assessments shall be assembled into “complementary, cumulative distribution functions” (CCDFs) that represent the probability of exceeding various levels of cumulative release caused by all significant processes and events. (b) Probability distributions for uncertain disposal system parameter values used in performance assessments shall be developed and documented in any compliance application. (c) Computational techniques, which draw random samples from across the entire range of the probability distributions developed pursuant to paragraph (b) of this section, shall be used in generating CCDFs and shall be documented in any compliance application. (d) … (f).
Ex. 2: Compliance Certification PA for WIPP

- Probability space \((\mathcal{A}, \mathbb{A}, p_A)\) for aleatory uncertainty
  - \(\mathcal{A} = \{a: a\text{ is }10^4 \text{ yr sequence of occurrences at WIPP}\}
  \[
  a: a = \begin{cases} 
  t_1, l_1, e_1, b_1, p_1, a_1, & \text{1st intrusion} \\
  \ldots \\
  t_n, l_n, e_n, b_n, p_n, a_n, & \text{nth intrusion} \\
  t_{min} \end{cases}
  \]
  where \(n\) is the number of drilling intrusions, \(t_i\) is the time (years) of the \(i\)th intrusion, \(l_i\) designates the location of the \(i\)th intrusion, \(e_i\) designates the penetration of an excavated or nonexcavated area by the \(i\)th intrusion, \(b_i\) designates whether or not the \(i\)th intrusion penetrates pressurized brine in the Castile Fm., \(p_i\) designates the plugging procedure used with the \(i\)th intrusion, \(a_i\) designates the type of waste penetrated by the \(i\)th intrusion, and \(t_{min}\) is the time at which potash mining occurs within the land withdrawal boundary.
  - \((\mathcal{A}, \mathbb{A}, p_A)\) derives from assumption that occurrence of drilling and mining characterized by Poisson processes

- Probability space \((\mathcal{E}, \mathbb{E}, p_E)\) for epistemic uncertainty
  - \(\mathcal{E} = \{e: e = [e_1, e_2, \ldots, e_{57}]\}
  - \((\mathcal{E}, \mathbb{E}, p_E)\) developed through expert review process
  - Implemented with 3 replicated LHSs of size 100 each

- Model \(y = f(a|e)\) representing physical behavior of system: Combination of models for physical processes (e.g., material deformation, two phase fluid flow, radionuclide transport) and interpolation/extrapolation procedures
Ex. 2: Comparison with EPA Release Requirements

- Results for RR1 = 1

\[
\begin{align*}
mRR_{1,0.05} & = 0.9553 \quad \text{mRR}_{1,0.5} = 0.9458 \\
mRR_{1,0.5} & = 0.9553 \
mRR_{1,0.05} & = 0.8741
\end{align*}
\]

\[
\begin{align*}
mRR_{1,0.05}/(mRR_{1,0.05} - mRR_{1,0.05}) & = 0.9458/(0.9458 - 0.8741) \\
& = 13.2 \\
mRR_{1,0.5}/(mRR_{1,0.5} - mRR_{1,0.05}) & = 0.9553/(0.9553 - 0.8741) \\
& = 11.8
\end{align*}
\]

- Analogous results for RR2 = 10
Ex. 3: Regulatory Requirements for YM Repository

- Many complex requirements
- This presentation considers following paraphrase of $10^4$ yr dose requirement

  The maximum expected dose to the reasonably maximally exposed individual (RMEI) over the first $10^4$ years following repository closure shall be less than 15 mrem/yr. (YM1)
Ex. 3: License Application PA for YM Repository

- Probability space \((\mathcal{A}, \mathbb{A}, p_A)\) for aleatory uncertainty
  - \(\mathcal{A} = \{a: a \text{ is 20,000 yr sequence of occurrences at YM repository}\}\)
    = \{a: a = [nEW, nED, nII, nIE, nSG, nSF, a_{EW}, a_{ED}, a_{ll}, a_{IE}, a_{SG}, a_{SF}]\}

  where
  - \(nEW\) = number of early WP failures,
  - \(a_{EW}\) = vector defining the \(nEW\) early WP failures,
  - \(nED\) = number of early DS failures,
  - \(a_{ED}\) = vector defining the \(nED\) early DS failures,
  - \(nII\) = number of igneous intrusive events,
  - \(a_{II}\) = vector defining the \(nII\) igneous intrusive events,
  - \(nIE\) = number of igneous eruptive events,
  - \(a_{IE}\) = vector defining the \(nIE\) igneous eruptive events,
  - \(nSG\) = number of seismic ground motion events,
  - \(a_{SG}\) = vector defining the \(nSG\) seismic ground motion events,
  - \(nSF\) = number of seismic fault displacement events,
  - \(a_{SF}\) = vector defining the \(nSF\) fault displacement events.

  - \((\mathcal{A}, \mathbb{A}, p_A)\) defined by distributions for elements of \(a\)

- Probability space \((\mathcal{E}, \mathbb{E}, p_E)\) for epistemic uncertainty
  - \(\mathcal{E} = \{e: e = [e_1, e_2, ..., e_{392}]\}\)
  - \((\mathcal{E}, \mathbb{E}, p_E)\) developed through expert review process
  - Implemented with LHS of size 300

- Model \(y = f(a|e)\) representing physical behavior of system: Combination of models for physical processes (e.g., heat flow, fluid flow, waste package degradation, radionuclide transport) and interpolation/extrapolation procedures
Ex 3: Comparison with NRC Dose Requirement

\[ mD_{mn} / (mD_{mn} - mD_{0.05}) \]
\[ = 14.76 / (14.76 - 14.29) \]
\[ = 31.40 \]

\[ mD_{0.5} / (mD_{0.5} - mD_{0.05}) \]
\[ = 14.88 / (14.88 - 14.29) \]
\[ = 25.22 \]
Additional Background/Information
(attached papers)


