The conceptual structure and computational organization of the 2008 total system performance assessment (TSPA) for the proposed high-level radioactive waste repository at Yucca Mountain, Nevada, are described. This analysis was carried out to support the License Application by the U.S. Department of Energy (DOE) to the U.S. Nuclear Regulatory Commission (NRC) for the indicated repository. In particular, the analysis was carried out to establish compliance with the postclosure requirements specified by the NRC in proposed 10 CFR Part 63. The requirements in 10 CFR Part 63 result in a performance assessment that involves three basic entities: (EN1) a characterization of the uncertainty in the occurrence of future events (e.g., igneous events, seismic events) that could affect the performance of the repository; (EN2) models for predicting the physical behavior and evolution of the repository (e.g., systems of ordinary and partial differential equations); and (EN3) a characterization of the uncertainty associated with analysis inputs that have fixed but imprecisely known values (e.g., the appropriate spatially-averaged value for a distribution coefficient). The designators aleatory and epistemic are commonly used for the uncertainties characterized by entities (EN1) and (EN3). The manner in which the preceding entities are defined and organized to produce the 2008 TSPA for the proposed Yucca Mountain repository are described.

I. INTRODUCTION

The appropriate disposal of radioactive waste from military and commercial activities is a challenge of national and international importance. As part of the solution to this challenge, a proposed deep geologic repository for high-level radioactive waste is under development by the U.S. Department of Energy (DOE) at Yucca Mountain (YM), Nevada. The development of the YM repository is the single most important radioactive waste disposal project currently being undertaken in the United States. The following presentation provides a description of the conceptual structure and computational organization of the 2008 total system performance assessment (TSPA) for the proposed YM repository.

II. REGULATORY BACKGROUND

As mandated in the Energy Policy Act of 1992, the U.S. Environmental Protection Agency (EPA) is required to promulgate public health and safety standards for radioactive material stored or disposed of in the YM repository; the U.S. Nuclear Regulatory Commission (NRC) is required to incorporate the EPA standards into licensing standards for the YM repository; and the DOE is required to show compliance with the NRC standards. The regulatory requirements for the YM repository that resulted from these mandates have two primary sources: (i) Public Health and Environmental Radiation Protection Standards for Yucca Mountain, NV; Final Rule (40 CFR Part 197), which has been promulgated by the EPA, and (ii) Disposal of High-Level Radioactive Wastes in a Proposed Geologic Repository at Yucca Mountain, Nevada; Final Rule (10 CFR Parts 2, 19, 20, etc.), which has been promulgated by the NRC. In turn, the DOE is required to carry out a performance assessment for the YM repository that satisfies the requirements specified in the preceding documents. In addition, the NRC has published the Yucca Mountain Review Plan; Final Report (YMRP) to guide assessing compliance with 10 CFR Parts 2, 19, 20, etc.

The initial EPA standard indicated above specified conditions that the YM repository was required to satisfy for the first 10 \(4\) yr after its closure. In a subsequent suit, it was ruled that the EPA did not follow guidance in a National Academy of Science (NAS) study as mandated by Congress in the Energy Policy Act of 1992. In particular, it was ruled that the EPA had failed to follow the guidance in the NAS study that the regulatory period for the YM repository should extend over the period of geologic stability at the facility site, which was suggested to be 10\(6\) yr. As a result, the initial regulation for the YM facility was remanded to the EPA for revision.

In response to this remand, the EPA published 40 CFR Part 197, Public Health and Environmental Radiation Protection Standards for Yucca Mountain, Nevada; Proposed Rule, which contained proposed revisions to the standards for the YM repository. Consistent with the EPA’s proposed revisions, the NRC published proposed 10 CFR Part 63, Implementation of a Dose Standard After 10,000 Years. The EPA’s and
NRC’s proposals in response to the remand left most of the requirements for the first $10^4$ yr after repository closure unchanged. However, new conditions were proposed for the time interval from $10^4$ yr through the period of geologic stability.

The overall structure of the YM 2008 TSPA derives from the individual protection standard specified by the EPA and the NRC. Specifically, the following standard is specified by the NRC (Ref. 8, p. 53319):

§ 63.311 Individual protection standard after permanent closure. (a) DOE must demonstrate, using performance assessment, that there is a reasonable expectation that the reasonably maximally exposed individual receives no more than the following annual dose from releases from the undisturbed Yucca Mountain disposal system: (1) 0.15 mSv (15 mrem) for 10,000 years following disposal; and (2) 3.5 mSv (350 mrem) after 10,000 years, but within the period of geologic stability. (b) DOE’s performance assessment must include all potential environmental pathways of radionuclide transport and exposure.

Except for minor differences in wording, the preceding standard is the same as the proposed standard specified by the EPA (Ref. 7, p. 49063).

In turn, the NRC gives the following guidance on implementing the preceding individual protection standard (Ref. 8, p. 53319):

§ 63.303 Implementation of Subpart L. (a) Compliance is based upon the arithmetic mean of the projected doses from DOE’s performance assessments for the period within 10,000 years after disposal for: (1) § 63.311(a); and (2) §§ 63.321(b)(1) and 63.331, if performance assessment is used to demonstrate compliance with either or both of these sections. (b) Compliance is based upon the median of the projected doses from DOE’s performance assessments for the period after 10,000 years of disposal and through the period of geologic stability for: (1) § 63.311(a); and (2) § 63.321(b)(2), if performance assessment is used to demonstrate compliance.

Again, the preceding is the same as the corresponding guidance given by the EPA (Ref. 7, p. 49063).

As indicated in (NRC1) and (NRC2), the NRC expects the determination of mean and median dose to the reasonably maximally exposed individual (RMEI) to be based on a detailed performance assessment. This expectation is further emphasized by the following statement in the YMRP (Ref. 4, p. 2.2-1):

Risk-Informed Review Process for Performance Assessment—The performance assessment quantifies repository performance, as a means of demonstrating compliance with the postclosure performance objectives at 10 CFR 63.113. The U.S. Department of Energy performance assessment is a systematic analysis that answers the triplet risk questions: what can happen; how likely is it to happen; and what are the consequences. (NRC3)

For convenience, the preceding questions can be represented by (Q1) “What can happen?”, (Q2) “How likely is it to happen?”, and (Q3) “What are the consequences if it does happen?”. The preceding questions provide the intuitive basis for the Kaplan/Garrick ordered triple representation for risk:

$$(S_i, pS_i, cS_i), i = 1, 2, \ldots, nS,$$

where (i) $S_i$ is a set of similar occurrences (i.e., the answer to Q1), (ii) $pS_i$ is the probability of $S_i$ (i.e., the answer to Q2), and (iii) $cS_i$ is a vector of consequences associated with $S_i$ (i.e., the answer to Q3). Further, the $S_i$ must be disjoint (i.e., $S_i \cap S_j = \emptyset$ for $i \neq j$); each $S_i$ must be sufficiently homogeneous to allow use of a single representative consequence vector $cS_i$; and $\cup S_i$ must contain all risk significant occurrences for the facility under consideration.

In addition, there is a fourth basic question that underlies the YM 2008 TSPA and, indeed, all complete performance assessments: (Q4) “What is the uncertainty in the answers to the initial three questions?”. The importance of answering this fourth question is emphasized in a number of statements by the NRC. For example:

For such long-term performance, what is required is reasonable expectation, making allowance for the time period, hazards, and uncertainties involved, that the outcome will conform with the objectives for postclosure performance for the geologic repository. Demonstrating compliance will involve the use of complex predictive models that are supported by limited data from field and laboratory tests, site-specific monitoring, and natural analog studies that may be supplemented with prevalent expert judgment. Compliance demonstrations should not exclude important parameters from assessments and analyses simply because they are difficult to precisely quantify to a high degree of confidence. The performance assessments and analyses should focus upon the full range of defensible and reasonable parameter distributions rather than on extreme physical situations and parameter values (Ref. 3, p. 55804). (NRC4)

Once again, although the criteria may be written in unqualified terms, the demonstration of compliance must take uncertainties and gaps in knowledge into account so that the Commission can make the specified finding with respect to paragraph (a)(2) of § 63.31 (Ref. 3, p. 55804). (NRC5)

Both the preceding statements clearly indicate that a reasonable treatment of uncertainty should be a fundamental part of a performance assessment used to support a licensing application for the YM repository.

III. CONCEPTUAL STRUCTURE

The YM 2008 TSPA was developed to satisfy requirements in 10 CFR Part 63 and has a structure that
involves three basic entities: (EN1) a characterization of the uncertainty in the occurrence of future events (e.g., igneous events, seismic events) that could affect the performance of the repository; (EN2) models for predicting the physical behavior and evolution of the repository (e.g., systems of ordinary and partial differential equations); and (EN3) a characterization of the uncertainty associated with analysis inputs that have fixed but imprecisely known values (e.g., the spatially-averaged value for a distribution coefficient). The designators aleatory and epistemic are commonly used for the uncertainties characterized by (EN1) and (EN3).

In the preceding, aleatory uncertainty is used in the designation of randomness in the possible future conditions that could affect the YM repository. In concept, each possible future at the YM repository can be represented by a vector

\[ a = [a_1, a_2, \ldots, a_n], \]

(2) where each \( a_j \) is a specific property of the future \( a \) (e.g., time of a seismic event, size of a seismic event, \( \ldots \)). In turn, a subset \( S \) of the set \( A \) of all possible values for \( a \) constitutes what is referred to as a scenario class in the YM 2008 TSPA. As part of the YM 2008 TSPA development, a probabilistic structure is imposed on the set \( A \). Formally, this corresponds to defining a probability space \( (A, \mathcal{A}, p_A) \) for aleatory uncertainty.

Then, \( \mathcal{A} \) is the set of all possible scenario classes, and \( p_A \) is the function that defines scenario class probability (i.e., scenario class \( S \) is an element of \( \mathcal{A} \) and \( p_A(S) \) is the probability of scenario class \( S \)). As discussed in more detail in Sect. VI, the set \( \mathcal{A} \) contains both disjoint and nondisjoint scenario classes. Formally, the probability space \( (A, \mathcal{A}, p_A) \) provides a characterization of aleatory uncertainty and constitutes the first of the three basic mathematical entities that underlie the determination of expected (i.e., mean) dose.

### TABLE I. Representation of Aleatory Uncertainty in the YM 2008 TSPA

<table>
<thead>
<tr>
<th>Individual Futures:</th>
<th>[ a = [n_{EW}, n_{ED}, n_{II}, n_{IE}, n_{SG}, n_{SF}, a_{EW}, a_{ED}, a_{II}, a_{IE}, a_{SG}, a_{SF}] ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Space for Aleatory Uncertainty:</td>
<td>[ A = { a : a = [n_{EW}, n_{ED}, n_{II}, n_{IE}, n_{SG}, n_{SF}, a_{EW}, a_{ED}, a_{II}, a_{IE}, a_{SG}, a_{SF}] } ]</td>
</tr>
<tr>
<td>Example Scenario Classes:</td>
<td>Nominal, ( A_N = { a : a \in A ) and ( n_{EW} = n_{ED} = n_{II} = n_{IE} = n_{SG} = n_{SF} = 0 } )</td>
</tr>
<tr>
<td></td>
<td>Early WP failure, ( A_{EW} = { a : a \in A ) and ( n_{EW} \geq 1 } ) ; Early DS failure, ( A_{ED} = { a : a \in A ) and ( n_{ED} \geq 1 } )</td>
</tr>
<tr>
<td></td>
<td>Igneous intrusive, ( A_{II} = { a : a \in A ) and ( n_{II} \geq 1 } ) ; Igneous eruptive, ( A_{IE} = { a : a \in A ) and ( n_{IE} \geq 1 } )</td>
</tr>
<tr>
<td></td>
<td>Seismic ground motion, ( A_{SG} = { a : a \in A ) and ( n_{SG} \geq 1 } ) ; Seismic fault displacement, ( A_{SF} = { a : a \in A ) and ( n_{SF} \geq 1 } )</td>
</tr>
<tr>
<td></td>
<td>Early failure, ( A_E = A_{EW} \cup A_{ED} ) ; Igneous, ( A_I = A_{II} \cup A_{IE} ) ; Seismic, ( A_S = A_{SG} \cup A_{SF} )</td>
</tr>
<tr>
<td>Scenario Class Probabilities:</td>
<td>( p_A(A_N) = ) probability of no disruptions of any kind</td>
</tr>
<tr>
<td></td>
<td>( p_A(A_{EW}) = ) probability of one or more early WP failures ; ( p_A(A_{ED}) = ) probability of one or more DS failures</td>
</tr>
<tr>
<td></td>
<td>( p_A(A_{II}) = ) probability of one or more II events ; ( p_A(A_{IE}) = ) probability of one or more IE events</td>
</tr>
<tr>
<td></td>
<td>( p_A(A_{SG}) = ) probability of one or more SG motion events ; ( p_A(A_{SF}) = ) probability of one or more SF displacement events</td>
</tr>
<tr>
<td></td>
<td>( p_A(A_E) = ) probability of one or more early failures ; ( p_A(A_I) = ) probability of one or more igneous events</td>
</tr>
<tr>
<td></td>
<td>( p_A(A_S) = ) probability of one or more seismic events</td>
</tr>
</tbody>
</table>
Although useful conceptually and notationally, the probability space \((A, \mathcal{A}, p_A)\) is never explicitly defined in the YM 2008 TSPA. Rather, the characterization of aleatory uncertainty enters the analysis through the definition of probability distributions for the individual elements of \(A\). Conceptually, the distributions for the elements of \(A\) lead to a distribution for \(A\) and an associated density function \(d_A(a)\). The nature of the probability space \((A, \mathcal{A}, p_A)\) in the context of the 2008 YM TSPA is summarized in Table I (see Ref. 11, App. J, for additional information).

The second of the three basic mathematical entities that underlie the determination of expected dose is a model that estimates dose to the RMEI. Formally, this model can be represented by the function

\[
D(\tau|A) = \text{dose to RMEI (mrem/yr) at time } \tau \text{ (yr) conditional on the occurrence of the future represented by } A.
\] 

(3)

Technically, \(D(\tau|A)\) is the committed 50 yr dose to the RMEI that results from radiation exposure incurred in a single year. In the computational implementation of the YM 2008 TSPA, \(D(\tau|A)\) is only one of the results calculated with the GoldSim program for the particular analysis configuration defined for the future \(A\). In practice, many results are calculated for \(A\) in addition to dose to the RMEI (see Ref. 11, Table K3-4). Thus, \(D(\tau|A)\) is part of a vector containing at least several thousand elements. For notational convenience, this paper presents the analysis for dose to the RMEI \(D(\tau|A)\); however, other YM 2008 TSPA results can be handled in exactly the same manner as described for dose. The general nature of \(D(\tau|A)\) is described in several following presentations\(^2\)\(^-\)\(^14\) and in more detail in Ref. 11.

The third of the three basic mathematical entities that underlie the determination of expected dose is a probabilistic characterization of epistemic uncertainty. Here, epistemic uncertainty refers to a lack of knowledge with respect to the appropriate value to use for a quantity that is assumed to have constant or fixed value in the context of a particular analysis. Specifically, epistemic uncertainty relates to a vector of the form

\[
e = [e_A, e_M] = [e_{A1}, e_{A2}, \ldots, e_{A,aE}; e_{M1}, e_{M2}, \ldots, e_{M,nME}]
\]

(4)

where

\[
e_A = [e_{A1}, e_{A2}, \ldots, e_{A,aE}]
\]

is a vector of epistemically uncertain quantities used in the characterization of aleatory uncertainty (e.g., a rate term that defines a Poisson process) and

\[
e_M = [e_{M1}, e_{M2}, \ldots, e_{M,nME}]
\]

is a vector of epistemically uncertain quantities used in the determination of dose (e.g., a distribution coefficient).

Epistemic uncertainty results in a set \(E\) of possible values for \(e\). In turn, probability is used to characterize the level of likelihood or credence that can be assigned to various subsets of \(E\). In concept, this leads to a probability space \((E, \mathcal{E}, p_E)\) for epistemic uncertainty. Like the probability space \((A, \mathcal{A}, p_A)\) for aleatory uncertainty, the probability space \((E, \mathcal{E}, p_E)\) for epistemic uncertainty is useful conceptually and notationally but is never explicitly defined in the YM 2008 TSPA. Rather, the characterization of epistemic uncertainty enters the analysis through the definition of probability distributions for the individual elements of \(e\). These distributions serve as mathematical summaries of all available information with respect to where the appropriate values for individual elements of \(e\) are located for use in the YM 2008 TSPA. Conceptually, the distributions for the elements of \(e\) lead to a distribution for \(e\) and an associated density function \(d_E(e)\). The nature of the probability space \((E, \mathcal{E}, p_E)\) in the context of the YM 2008 TSPA is indicated in Table II (see Ref. 11, Tables K3-1, K3-2, K3-3, for additional information).

TABLE II. Examples of the \(nE = 392\) Epistemically Uncertain Variables Considered in the YM 2008 TSPA

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Distribution</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>IGRATE</td>
<td>Frequency of intersection of the repository footprint by a volcanic event (yr⁻¹). Distribution: Piecewise uniform. Range: 0 to 7.76E-07.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INFIL</td>
<td>Pointer variable for determining infiltration conditions: 10th, 30th, 50th or 90th percentile infiltration scenario (dimensionless). Distribution: Discrete. Range: [1, 2, 3, 4].</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MICPU239</td>
<td>Groundwater biosphere dose conversion factor (BDCF) for ²³⁹Pu in modern interglacial climate ((Sv/year)/(Bq/m³)). Distribution: Discrete. Range: 3.49E-07 to 2.93E-06.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SZFISPVO</td>
<td>Flowing interval spacing in fractured volcanic units (m). Distribution: Piecewise uniform. Range: 1.86 to 80.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
IV. EXPECTED DOSE, MEAN DOSE, MEDIAN DOSE

Now that the characterization of epistemic uncertainty has been introduced, the notations used to represent aleatory uncertainty and dose need to be expanded. Because the representation of aleatory uncertainty depends on elements of the vector \( \mathbf{e}_A \), each possible value for \( \mathbf{e}_A \) could lead to a different probability space \( (\mathcal{A}, \mathcal{A}, p_A) \) for aleatory uncertainty. For notational convenience, this dependence will be indicated by representing the density function associated with aleatory uncertainty by \( d_A(\mathbf{a}|\mathbf{e}_A) \). Similarly, the determination of dose depends on elements of the vector \( \mathbf{e}_M \), with each possible value for \( \mathbf{e}_M \) potentially leading to different dose results. For notational convenience, this dependence will be indicated by representing the dose function by \( D(\tau|\mathbf{a}, \mathbf{e}_M) \).

The probability space \( (\mathcal{A}, \mathcal{A}, p_A) \) for aleatory uncertainty characterized by the density function \( d_A(\mathbf{a}|\mathbf{e}_A) \), the dose function \( D(\tau|\mathbf{a}, \mathbf{e}_M) \), and the probability space \( (\mathcal{E}, \mathbb{E}, p_E) \) for epistemic uncertainty characterized by the density function \( d_E(\mathbf{e}) \) constitute the three basic parts of the YM 2008 TSPA that come together in the determination of expected dose to the RMEI and the uncertainty in expected dose to the RMEI. Specifically, the expected value for dose at time \( \tau \) conditional on a specific element \( \mathbf{e} = [\mathbf{e}_A, \mathbf{e}_M] \) of \( \mathcal{E} \) is given by

\[
\bar{D}(\tau|\mathbf{e}) = E_A[D(\tau|\mathbf{a}, \mathbf{e}_M)|\mathbf{e}_A] = \int_{\mathcal{A}} D(\tau|\mathbf{a}, \mathbf{e}_M) d_A(\mathbf{a}|\mathbf{e}_A) dA, \tag{5}
\]

where \( E_A[D(\tau|\mathbf{a}, \mathbf{e}_M)|\mathbf{e}_A] \) denotes expectation over aleatory uncertainty.

In turn, the uncertainty associated with the estimation of \( \bar{D}(\tau|\mathbf{e}) \) can be determined from the properties of the probability space \( (\mathcal{E}, \mathbb{E}, p_E) \) for epistemic uncertainty. In particular, the cumulative distribution function (CDF) for \( \bar{D}(\tau|\mathbf{e}) \) and the expected value for \( \bar{D}(\tau|\mathbf{e}) \) that derive from epistemic uncertainty are given by

\[
p_E[\bar{D}(\tau|\mathbf{e}) \leq D] = \int_{\mathcal{E}} \tilde{\mathcal{E}}_D[\bar{D}(\tau|\mathbf{e})] d_E(\mathbf{e}) dE = \int_{\mathcal{E}} \tilde{\mathcal{E}}_D[\int_{\mathcal{A}} D(\tau|\mathbf{a}, \mathbf{e}_M) d_A(\mathbf{a}|\mathbf{e}_A) dA] d_E(\mathbf{e}) dE, \tag{6}
\]

where

\[
\bar{D}(\tau) = E_E[\bar{D}(\tau|\mathbf{e})] = \int_{\mathcal{E}} \bar{D}(\tau|\mathbf{e}) d_E(\mathbf{e}) dE, \tag{7}
\]

respectively, where

\[
\tilde{\mathcal{E}}_D[\bar{D}(\tau|\mathbf{e})] = \begin{cases} 1 & \text{if } \bar{D}(\tau|\mathbf{e}) \leq D \\ 0 & \text{if } \bar{D}(\tau|\mathbf{e}) > D \end{cases}
\]

and \( E_E[\bar{D}(\tau|\mathbf{e})] \) denotes expectation over epistemic uncertainty.

The individual grey curves in Fig. 1 correspond to expected doses \( \bar{D}(\tau|\mathbf{e}) \) as defined in Eq. (5). The totality of the grey curves provides a display of the uncertainty in \( \bar{D}(\tau|\mathbf{e}) \) that derives from the uncertainty in \( \mathbf{e} \). The red curve in Fig. 1 corresponds to the mean dose \( \bar{D}(\tau) \) defined in Eq. (7) and used in comparisons with the \( 10^4 \) yr standard as specified in Quotes (NRC1) and (NRC2). Specifically, \( \bar{D}(\tau) \) is the expected value for \( \bar{D}(\tau|\mathbf{e}) \) over the epistemic uncertainty associated with \( \mathbf{e} \).

The value of \( D \) for which

\[
q = p_E[\bar{D}(\tau|\mathbf{e}) \leq D] = \int_{\mathcal{E}} \tilde{\mathcal{E}}_D[\bar{D}(\tau|\mathbf{e})] d_E(\mathbf{e}) dE \tag{8}
\]

defines the \( q \) quantile (e.g., \( q = 0.05, 0.5, 0.95 \)) for the distribution of expected dose over epistemically uncertain analysis inputs. For notational purposes, the value of \( D \) corresponding to the \( q \) quantile of \( \bar{D}(\tau|\mathbf{e}) \) defined in Eq. (8) will be represented by \( Q_{E,q}[\bar{D}(\tau|\mathbf{e})] \). The blue curve in Fig. 1 corresponds to the median dose \( Q_{E,0.5}[\bar{D}(\tau|\mathbf{e})] \) defined in Eq. (8) for \( q = 0.5 \) and used in comparisons with the proposed post \( 10^4 \) yr standard as specified in Quotes (NRC1) and (NRC2).

Fig. 1 Expected, mean and median curves for dose to the RMEI

where

\[
\tilde{\mathcal{E}}_D[\bar{D}(\tau|\mathbf{e})] = \begin{cases} 1 & \text{if } \bar{D}(\tau|\mathbf{e}) \leq D \\ 0 & \text{if } \bar{D}(\tau|\mathbf{e}) > D \end{cases}
\]

and

\[
\bar{D}(\tau) = E_E[\bar{D}(\tau|\mathbf{e})] = \int_{\mathcal{E}} \bar{D}(\tau|\mathbf{e}) d_E(\mathbf{e}) dE, \tag{7}
\]

respectively, where

\[
\tilde{\mathcal{E}}_D[\bar{D}(\tau|\mathbf{e})] = \begin{cases} 1 & \text{if } \bar{D}(\tau|\mathbf{e}) \leq D \\ 0 & \text{if } \bar{D}(\tau|\mathbf{e}) > D \end{cases}
\]

and \( E_E[\bar{D}(\tau|\mathbf{e})] \) denotes expectation over epistemic uncertainty.

The individual grey curves in Fig. 1 correspond to expected doses \( \bar{D}(\tau|\mathbf{e}) \) as defined in Eq. (5). The totality of the grey curves provides a display of the uncertainty in \( \bar{D}(\tau|\mathbf{e}) \) that derives from the uncertainty in \( \mathbf{e} \). The red curve in Fig. 1 corresponds to the mean dose \( \bar{D}(\tau) \) defined in Eq. (7) and used in comparisons with the \( 10^4 \) yr standard as specified in Quotes (NRC1) and (NRC2). Specifically, \( \bar{D}(\tau) \) is the expected value for \( \bar{D}(\tau|\mathbf{e}) \) over the epistemic uncertainty associated with \( \mathbf{e} \).
TABLE III. Decomposition of Expected Dose $\mathcal{D}(\tau \mid \mathbf{e})$ into Expected Incremental Doses $\mathcal{D}_C(\tau \mid \mathbf{e})$ from Individual Scenario Classes

\[
\mathcal{D}(\tau \mid \mathbf{e}) = \int_{\mathcal{A}} D(\tau \mid \mathbf{a}, \mathbf{e}_M) d_A(\mathbf{a} \mid \mathbf{e}_A) dA \\
\leq \int_{\mathcal{A}} \left\{ D_N(\tau \mid \mathbf{a}_N, \mathbf{e}_M) + \sum_{C \in \mathcal{MC}} D_C(\tau \mid \mathbf{a}, \mathbf{e}_M) \right\} d_A(\mathbf{a} \mid \mathbf{e}_A) dA \\
= D_N(\tau \mid \mathbf{a}_N, \mathbf{e}_M) + \sum_{C \in \mathcal{MC}} \int_{\mathcal{A}} D_C(\tau \mid \mathbf{a}, \mathbf{e}_M) d_A(\mathbf{a} \mid \mathbf{e}_A) dA \\
= D_N(\tau \mid \mathbf{a}_N, \mathbf{e}_M) + \sum_{C \in \mathcal{MC}} \mathcal{D}_C(\tau \mid \mathbf{e})
\]

where $\mathbf{a}_N$ corresponds to the single future associated with the nominal scenario class $\mathcal{A}_N$ in which no disruptions of any kind occur, $D_N(\tau \mid \mathbf{a}_N, \mathbf{e}_M)$ is the dose to the RMEI that results solely from processes associated with the nominal scenario class, and $D_C(\tau \mid \mathbf{a}, \mathbf{e}_M)$ is the incremental dose to the RMEI that results solely from the effects of the disruptions that result in the future $\mathbf{a}$ being an element of the scenario class (i.e., set) $\mathcal{A}_C$.

V. COMPUTATIONAL IMPLEMENTATION

Evaluation of expected, mean and median doses as described in the preceding section presents two numerical challenges. First, it is necessary to evaluate integrals over the set $\mathcal{A}$ to obtain expected doses over aleatory uncertainty. Second, it is necessary to evaluate integrals over the set $\mathcal{E}$ to obtain mean and median doses over aleatory and epistemic uncertainty.

Evaluation of integrals over the set $\mathcal{A}$ is considered first. These evaluations are accomplished under the assumption that there are no synergisms between the effects of the disruptions associated with the individual scenario classes that have a significant effect on the expected dose $\mathcal{D}(\tau \mid \mathbf{e})$. As a result and with the assumption that nominal process releases occur for all scenario classes, $\mathcal{D}(\tau \mid \mathbf{e})$ can be approximated as indicated in Table III. Example derivations of how the use of disjoint scenario classes to calculate expected dose $\mathcal{D}(\tau \mid \mathbf{e})$ in combination with the no significant synergisms assumption leads to the relationships Table III are presented in Ref. 16.

Given the decomposition in Table III, $\mathcal{D}(\tau \mid \mathbf{e})$ can be approximated by (i) approximating $D_N(\tau \mid \mathbf{a}_N, \mathbf{e}_M)$ and individually approximating the integrals defining the expected incremental doses $\mathcal{D}_C(\tau \mid \mathbf{e})$ as indicated in Table IV (see Ref. 11, App. J, for additional details) and then (ii) adding these approximations to obtain an approximation to $\mathcal{D}(\tau \mid \mathbf{e})$.

TABLE IV. Integration Procedures Used to Obtain Expected Incremental Dose $\mathcal{D}_C(\tau \mid \mathbf{e})$ for Individual Scenario Classes in the YM 2008 TSPA

<table>
<thead>
<tr>
<th>Scenario Class</th>
<th>Integration Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early WP and DS Failures</td>
<td>Summation of probabilistically weighted results for individual failures</td>
</tr>
<tr>
<td>Igneous Intrusive Events</td>
<td>Quadrature procedure</td>
</tr>
<tr>
<td>Igneous Eruptive Events</td>
<td>Combined Quadrature/Monte Carlo procedure</td>
</tr>
<tr>
<td>Seismic Ground Motion Events</td>
<td>Quadrature procedure</td>
</tr>
<tr>
<td>Seismic Fault Displacement Events</td>
<td>Quadrature procedure</td>
</tr>
</tbody>
</table>

Nominal Conditions: $D_N(\tau \mid \mathbf{e})$
- Always zero for $[0, 2 \times 10^4 \text{ yr}]$ in YM 2008 TSPA
- Combined with seismic ground motion for $[0, 10^6 \text{ yr}]$

Early WP and DS Failures: $\mathcal{D}_{EW}(\tau \mid \mathbf{e}), \mathcal{D}_{ED}(\tau \mid \mathbf{e})$
- Summation of probabilistically weighted results for individual failures

Igneous Intrusive Events: $\mathcal{D}_{II}(\tau \mid \mathbf{e})$
- Quadrature procedure

Igneous Eruptive Events
- Combined Quadrature/Monte Carlo procedure

Seismic Ground Motion Events: $\mathcal{D}_{SG}(\tau \mid \mathbf{e})$
- Quadrature procedure for $[0, 2 \times 10^4 \text{ yr}]$
- Monte Carlo procedure for $[0, 10^6 \text{ yr}]$

Seismic Fault Displacement Events: $\mathcal{D}_{SF}(\tau \mid \mathbf{e})$
- Quadrature procedure
The mean dose $\overline{D}(\tau)$ and the median dose $Q_{E,0.5}[\overline{D}(\tau|\mathbf{e})]$ are defined by integrals over the set $\mathcal{E}$ of epistemically uncertain analysis inputs as indicated in Eqs. (7) and (8). In the YM 2008 TSPA, these integrals are approximated with use of a Latin hypercube sample (LHS)

$$\mathbf{e}_i = [\mathbf{e}_{Al}, \mathbf{e}_{Ml}], i = 1, 2, \ldots, nLHS,$$

of size $nLHS = 300$ generated in consistency with the definition of the probability space $(\mathcal{E}, \mathcal{B}, P_E)$ (i.e., in consistency with the distributions defined for the individual elements of $\mathbf{e}$). Then, $\overline{D}(\tau)$ and $p_E[\overline{D}(\tau|\mathbf{e}) \leq D]$ are approximated by

$$\overline{D}(\tau) \cong \frac{\sum_{i=1}^{nLHS} \overline{D}(\tau|\mathbf{e}_i)}{nLHS}$$

and

$$p_E[\overline{D}(\tau|\mathbf{e}) \leq D] \cong \frac{\sum_{i=1}^{nLHS} \overline{D}(\tau|\mathbf{e}_i)}{nLHS},$$

respectively. Further, this sample can be used in a numerical determination of the quantiles $Q_{E,\alpha}[\overline{D}(\tau|\mathbf{e})]$ for $\overline{D}(\tau|\mathbf{e})$ defined in Eq. (8). Analogous approximations to mean and median doses over epistemic uncertainty also exist for the individual scenario classes.

VI. DISJOINT AND NONDISJOINT SCENARIO CLASSES

As used in this presentation, a scenario class is any element of the set $\mathcal{A}$ appearing in the formal definition of the probability space $(\mathcal{A}, \mathcal{B}, P_A)$ for aleatory uncertainty. Specifically, a scenario class is any subset $S$ of the set $\mathcal{A}$ of possible futures (see Table I) for which a probability $P_A(S)$ can be defined. This definition, which is consistent with the formal development of probability, allows for both disjoint and nondisjoint scenario classes. Consistent with this, both disjoint and nondisjoint scenario classes have significant roles in the YM 2008 TSPA.

As recognized by the NRC in the following statement from the YMRP (Ref. 4, p. 2.2-133), the calculation of expected dose to the RMEI has a conceptual basis that involves the use of disjoint scenario classes: “The occurrence of scenario classes, included in the calculating the annual dose, sum to one.” This statement is consistent with the approximation of the expected dose $\overline{D}(\tau|\mathbf{e})$ defined in Eq. (5) by

$$\overline{D}(\tau|\mathbf{e}) \cong \sum_{i=1}^{nS} D(\tau|\mathbf{a}_i, \mathbf{e}_M) P_A(S_i|\mathbf{e}_A),$$

where the $S_i$ are elements of $\mathcal{A}$ (i.e., subsets of $\mathcal{A}$), $S_i \cap S_j = \emptyset$ if $i \neq j$, $\cup_i S_i = \mathcal{A}$, $\mathbf{a}_i \in S_i$, and $P_A(S_i|\mathbf{e}_A)$ is the probability of $S_i$. The preceding approximation to $\overline{D}(\tau|\mathbf{e})$ corresponds to an expected value calculation in the context of the ordered triplet representation for risk $(S_i, pS_i, cS_i)$, $i = 1, 2, \ldots, nS$, in Eq. (1). Specifically, the sets $S_i$ are the same, $P_A(S_i|\mathbf{e}_A)$ corresponds to $pS_i$, and $D(\tau|\mathbf{a}_i, \mathbf{e}_M)$ corresponds to $cS_i$.

As indicated in Eq. (12), the calculation of expected dose to the RMEI in the YM 2008 TSPA can be formally based on the consideration of disjoint scenario classes with probabilities that sum to one. However, in computational practice, the number of disjoint scenario classes required for the sum in Eq. (12) to be a reasonable approximation to $\overline{D}(\tau|\mathbf{e})$ is both large and difficult to determine (e.g., see Ref. 15). For this reason and with described justification (Ref. 11, App. J), the YM 2008 TSPA approximates $\overline{D}(\tau|\mathbf{e})$ on the basis of the no significant synergisms decomposition indicated in Table III. This decomposition involves the nondisjoint scenario classes $\mathcal{A}_C$, $C = EW, ED, II, IE, SG, SF$, appearing in Tables I and III. However, the starting integral that defines $\overline{D}(\tau|\mathbf{e})$ in Eq. (5) is predicated on the concept of disjoint scenario classes. In particular, the correct place to check for conservation of probability in the determination of $\overline{D}(\tau|\mathbf{e})$ is in the integral definition of $\overline{D}(\tau|\mathbf{e})$ in Eq. (5) rather than after the no significant synergisms assumption has been implemented at the end of Table III. This decomposition is very beneficial because implementing integrals over the sets $\mathcal{A}_C$ (or modeling cases as they are sometimes called; see Table IV) is much easier that implementing an integral over the set $\mathcal{A}$. This decomposition also facilitates informative uncertainty and sensitivity analyses of the form presented in Refs. 17 and 18 and in more detail in Apps. J and K of Ref. 11.

In addition, when scenario class probabilities are requested, it is likely that the desired probabilities are for the in the YM 2008 TSPA.

As recognized by the NRC in the following statement from the YMRP (Ref. 4, p. 2.2-133), the calculation of expected dose to the RMEI has a conceptual basis that involves the use of disjoint scenario classes: “The occurrence of scenario classes, included in the calculating the annual dose, sum to one.” This statement is consistent with the approximation of the expected dose $\overline{D}(\tau|\mathbf{e})$ defined in Eq. (5) by

$$\overline{D}(\tau|\mathbf{e}) \cong \sum_{i=1}^{nS} D(\tau|\mathbf{a}_i, \mathbf{e}_M) P_A(S_i|\mathbf{e}_A),$$

where the $S_i$ are elements of $\mathcal{A}$ (i.e., subsets of $\mathcal{A}$), $S_i \cap S_j = \emptyset$ if $i \neq j$, $\cup_i S_i = \mathcal{A}$, $\mathbf{a}_i \in S_i$, and $P_A(S_i|\mathbf{e}_A)$ is the probability of $S_i$. The preceding approximation to $\overline{D}(\tau|\mathbf{e})$ corresponds to an expected value calculation in the context of the ordered triplet representation for risk $(S_i, pS_i, cS_i)$, $i = 1, 2, \ldots, nS$, in Eq. (1). Specifically, the sets $S_i$ are the same, $P_A(S_i|\mathbf{e}_A)$ corresponds to $pS_i$, and $D(\tau|\mathbf{a}_i, \mathbf{e}_M)$ corresponds to $cS_i$.

As indicated in Eq. (12), the calculation of expected dose to the RMEI in the YM 2008 TSPA can be formally based on the consideration of disjoint scenario classes with probabilities that sum to one. However, in computational practice, the number of disjoint scenario classes required for the sum in Eq. (12) to be a reasonable approximation to $\overline{D}(\tau|\mathbf{e})$ is both large and difficult to determine (e.g., see Ref. 15). For this reason and with described justification (Ref. 11, App. J), the YM 2008 TSPA approximates $\overline{D}(\tau|\mathbf{e})$ on the basis of the no significant synergisms decomposition indicated in Table III. This decomposition involves the nondisjoint scenario classes $\mathcal{A}_C$, $C = EW, ED, II, IE, SG, SF$, appearing in Tables I and III. However, the starting integral that defines $\overline{D}(\tau|\mathbf{e})$ in Eq. (5) is predicated on the concept of disjoint scenario classes. In particular, the correct place to check for conservation of probability in the determination of $\overline{D}(\tau|\mathbf{e})$ is in the integral definition of $\overline{D}(\tau|\mathbf{e})$ in Eq. (5) rather than after the no significant synergisms assumption has been implemented at the end of Table III. This decomposition is very beneficial because implementing integrals over the sets $\mathcal{A}_C$ (or modeling cases as they are sometimes called; see Table IV) is much easier that implementing an integral over the set $\mathcal{A}$. This decomposition also facilitates informative uncertainty and sensitivity analyses of the form presented in Refs. 17 and 18 and in more detail in Apps. J and K of Ref. 11.

In addition, when scenario class probabilities are requested, it is likely that the desired probabilities are for the nondisjoint scenario classes $\mathcal{A}_C$, $C = EW, ED, II, IE, SG, SF$, or possibly some other collection of nondisjoint scenario classes. In particular, it is probabilities for the nondisjoint scenario classes $\mathcal{A}_C$ that are presented in App. J of Ref. 11. For example, if probability of early WP failure is under consideration, then most likely $p_A(\mathcal{A}_{EW}|\mathbf{e}_A)$ rather than $p_A(\mathcal{A}_{EW}|\mathbf{e}_A)$ is the probability of interest. Specifically, the $p_A(\mathcal{A}_{EW}|\mathbf{e}_A)$ is the probability that one or more early WP failures occur while $p_A(\mathcal{A}_{EW}|\mathbf{e}_A)$ is the probability that two or more early WP failures occur while $p_A(\mathcal{A}_{EW}|\mathbf{e}_A)$ is the probability that three or more early WP failures occur while...
nIE = nSG = nSF = 0 | (e, j) is the probability that one or more early failures occur and also that no other failures of any other type occur; this latter probability is significantly affected by the indicated nonoccurrence assumptions and effectively provides no information on the likelihood of early WP failures.

VII. SUMMARY

As described, the conceptual and computational structure of the YM 2008 TSPA is based on three basic entities: (EN1) a characterization of the uncertainty in the occurrence of future events that could affect the performance of the repository (i.e., a probability space (A, \( \hat{A}, P_A \)) characterizing aleatory uncertainty), (EN2) models for predicting the physical behavior and evolution of the repository system (i.e., a very complex function \( D(\|a, e_M\|) \) that predicts dose to the RMEI and a large number of additional analysis results), and (EN3) a characterization of the uncertainty associated with analysis inputs that have fixed but imprecisely known values (i.e., a probability space \((E, E, p_E)\) characterizing epistemic uncertainty).

This paper summarizes the first presentation in a special session intended to provide an overview on the YM 2008 TSPA. Following presentations in the session provide summaries of (i) the development and use of the models that collectively constitute the function \( D(\|a, e_M\|) \) that predicts dose to the RMEI and a large number of additional analysis results), and (EN3) a characterization of the uncertainty associated with analysis inputs that have fixed but imprecisely known values (i.e., a probability space \((E, E, p_E)\) characterizing epistemic uncertainty).

(i) the performance of uncertainty and sensitivity analyses for physical processes based on \( D(\|a, e_M\|) \) and the characterization of epistemic uncertainty provided by \((E, E, p_E)\), (ii) the performance of uncertainty and sensitivity analyses for physical processes based on \( D(\|a, e_M\|) \) and the characterization of epistemic uncertainty provided by \((E, E, p_E)\), (iii) the performance of uncertainty and sensitivity analyses for expected dose to the RMEI based on the characterization of aleatory uncertainty provided by \((A, \hat{A}, P_A)\), (iv) a summary of the YM 2008 TSPA in the context of the regulatory requirements specified by the NRC in 10 CFR Part 63.

Additional and more detailed information on the YM 2008 PA is available in a detailed analysis report and in the references cited in this report.

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