Stat 345-002 Fall 2004 Name: _____

FINAL EXAMINATION (75 points)

JUSTIFY YOUR ANSWERS AND SHOW ALL OF YOUR WORK FOR PARTIAL CREDIT. This is a closed book, closed notes, closed neighbor exam. You may use a calculator and two pages (8.5x11") of notes. The last page of your exam is scratch paper which you may tear off and use and discard. DO NOT UNSTAPLE THE EXAM.

- 1. Let the random variable X have E(X) = 2 and Var(X) = 4. Let the random variable Y have E(Y) = 2 and Var(Y) = 9. Suppose that X and Y are independent random variables.
 - (a) (2 pts) Let Z = 3X 2Y. Find the expected value of the random variable Z.

(b) (3 pts) Let W = 3X - 2Y + 5. Find the variance of the random variable W.

(c) (3 pts) If X and Y are Normally distributed random variables, which is the distribution of Z in part (a)? (name also the parameters of the distribution).

2. Let X denote the number of times a certain numerical control machine will malfunction: 1 or 2 on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as:

		X	
	$f_{XY}(x,y)$	1	2
Y	1	0.3	0.45
	2	0.1	0.15

(a) (2 pts) Find the marginal pmf of X.

(b) (2 pts) Find E(X).

(c) (2 pts) Find Var(X).

(d) (2 pts) Find P(Y = 1 | X = 2).

(e) (2 pts) Find E(XY).

(f) (2 pts) Find COV(X, Y).

(g) (2 pts) Find CORR(X, Y).

(h) (2 pts) Describe the strength and direction of the linear relationship between X and Y.

- 3. A certain type of storage battery lasts, on average, 3.0 years with a standard deviation of 0.5 year. Assuming that the battery lives are normally distributed, find the probability that a given battery will last
 - (a) (3 pts) less than 2.3 years;

(b) (3 pts) between 2.25 and 3.75 years.

(c) (3 pts) What is the number of years for which the lifetime of a battery is exceeded with 85% probability?

- 4. A drug manufacturer claims that a certain drug cures a blood disease on the average 80% of the time. To check the claim, government testers used the drug on a sample of 100 individuals and decided to accept the claim if 75 or more are cured. If X is the number of persons that are cured from this sample of 100 individuals,
 - (a) (2 pts) what is the distribution of X? (name it and give the values of the parameters).

(b) (2 pts) What are the mean and variance of X?

(c) (4 pts) What is the probability that the claim will be rejected when the cure probability is, in fact, 0.8?

5. The random variable X, representing the number of cherries in a cherry puff, has the following probability distribution:

x	4	5	6	7
$f_X(x)$	0.2	0.4	0.3	0.1

(a) (3 pts) Find the mean and the variance of X.

(b) (2 pts) Find the mean and the variance of the sample mean \bar{X} for random samples of 36 cherry puffs.

(c) (2 pts) What is the approximate distribution of \bar{X} ?

(d) (3 pts) Find the probability that the average number of cherries, \bar{X} , of 36 cherry puffs will be less than 5.5.

- 6. A random sample of 25 bottles of buffered aspirin contain, on average, 325.05 mg of aspirin with a standard deviation of 0.5. Assume that the aspirin content is normally distributed.
 - (a) (4 pts) Find a 95% confidence interval for the true mean of a spirin content of these bottles.

(b) (3 pts) Provide an interpretation of your confidence interval in (a).

- 7. The pdf of a continuous random variable X is given by f(x) = cx(1+x); 0 < x < 2 and 0 elsewhere.
 - (a) (2 pts) Find c so that f(x) is a valid density function.

(b) (2 pts) Find P(X > 1).

(c) (3 pts) Find $E(X^2 + 2)$.

(d) (3pts) Find the cdf of X (be careful to define the cdf for all values of x).

- 8. It is suggested that 75 % of adults think the space program is a good investment for the country. According to a report in the *Roanoke Times and World-News*, 1066 out of 1600 adults polled by telephone said the space program is a good investment for the country.
 - (a) (4 pts) Find a 95 % confidence interval for the proportion of American adults who think the space program is a good investment for the country.

(b) (3 pts) Based on your interval, can we support the hypothesis of a 75% of adults that think the space program is a good investment for the country? Justify your answer.